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Reflection and refraction of *SH*-waves at a corrugated interface between two laterally and vertically heterogeneous viscoelastic solid half-spaces

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Abstract

Reflection and transmission coefficients due to incident plane *SH*-waves at a corrugated interface between two isotropic, laterally and vertically heterogeneous visco-elastic solid half spaces are obtained. The density and complex rigidity of each medium are considered to vary along horizontal and vertical directions. Closed form expressions of reflection and transmission coefficients are derived using Rayleigh's method of approximation. These coefficients are found to be the function of corrugation, heterogeneity, angle of incidence, angle between propagation and attenuation vectors and visco-elasticity of the media. Numerical computations are made for a specific model to study the nature of dependence of these coefficients. Variations of reflection and transmission coefficients for the first order of approximation of the corrugation versus angle of incidence, corrugation and angle between propagation and attenuation vectors are computed and depicted graphically. Comparison is made between these coefficients in viscoelastic media and in uniform elastic media. The problems investigated earlier by Asano [Bull. Earthq. Res. Inst. 38 (1960) 177], Singh et al. [Acta Geophys. Pol. XXVI (1978) 209], Kaushik and Chopra [Geophys. Res. Bull. 18 (1980) 111] and Gupta [Geophys. Trans. 33 (1987) 89] have been reduced as particular cases.

Keywords: *SH*-wave; Rayleigh's method of approximation; Reflection; Transmission; Viscoelastic; Laterally and vertically heterogeneous; Corrugation

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1. Introduction

Mathematical treatment of propagation of seismic waves and their reflection and refraction from various types of surfaces are widely applied in the problems of seismology and seismic explorations. Reflection seismology is a tool for investigating the internal structure of the earth as well as for exploration of valuable materials buried under the earth surface. Many recent studies based on the body and surface wave analysis have led to the conclusion that there are significant lateral and vertical variations in the elastic properties of the earth medium. Also, there are several types of discontinuities present in the earth. These discontinuities within the earth may be irregular in nature and cannot be considered as perfectly plane interface. It is, therefore, necessary to take into account the roughness or irregularity of the interface, while studying the problems of reflection and refraction of seismic waves. These reflected and refracted waves carry a lot of information with them which are helpful in studying the character of the material present in the earth among several other important informations. Therefore, the study of reflection and transmission of elastic/seismic waves at the corrugated interface is of great practical importance in theoretical as well as in observational seismology.

The problems of wave propagation and their reflection and refraction from plane boundaries have been studied by several researchers in the past and have appeared in the open literature e.g. *see* Ewing et al. (1957), Brekhovskikh (1960), Achenbach (1976), Aki and Richards (1980), Ben-Menahem and Singh (1981), Sheriff and Geldart (1982), Lay and Wallace (1995) and Udias (1999). Knowledge of reflection and transmission coefficients of *SH*-waves in a layered visco-elastic homogeneous and inhomogeneous medium is essential for calculations of amplitudes of various seismic signals. The general theory of visco-elasticity describes the linear behaviour of both elastic and anelastic materials and provides the basis for describing the attenuation of seismic waves due to anelasticity. Cooper and Reiss (1966), Cooper (1967), Buchen (1971a,b) and Borcherdt (1973a,b, 1977), Borcherdt and Wenneberg (1985), Borcherdt et al. (1986), Borcherdt (1989) and Romeo (2003) investigated various problems of wave propagation in visco-elasticity. Kaushik and Chopra (1980, 1981, 1983, 1984) studied the problems of reflection and transmission of *SH*-waves at a plane interface by incorporating different nature of elastic properties in linear visco-elastic half spaces and discussed their effects on amplitude ratios of reflected and refracted waves. They have also discussed the existence of critical angle of the incident wave for some cases. Gogna and Chander (1985) discussed reflection and transmission of *SH*-waves at a plane interface between anisotropic inhomogeneous elastic and visco-elastic half-spaces.

In all the above investigations, reflection and transmission of *SH*-waves were considered at a plane interface. Asano (1960, 1961, 1966), Gupta (1987), Tomar et al. (2002), Kumar et al. (2003) and Tomar and Kaur (2003) attempted problems of reflection and refraction of elastic waves at a corrugated interface between two different elastic solid half spaces using Rayleigh's method by incorporating the effects of elastic and anelastic material properties. Abubakar (1962a,b,c) used method of small perturbations to investigate the problems of reflection and refraction of elastic waves at rough boundaries. In the present problem, using Rayleigh's method of approximation, we have attempted a problem of reflection and transmission of *SH*-waves at a corrugated interface between two horizontally and vertically heterogeneous visco-elastic solid half spaces. The approach is based on a linear model of visco-elasticity and on a Fourier expansion of the function which describes the interface's shape. An exponential law is considered to account for material inhomogeneity. The continuity conditions for displacement and traction at the interface are exploited to obtain the zeroth and the first order approximations to obtain the reflection and transmission coefficients. Results of Kaushik and Chopra (1980, 1984), Gupta (1987), Singh et al. (1978) and Asano (1960) have been obtained as particular cases of the present problem.

2. Formulation and theory

We consider a corrugated interface separating the two horizontally and vertically inhomogeneous visco-elastic solid half spaces with different elastic properties. Let x–y plane be horizontal and z-axis be pointing vertically downwards. The geometry of the problem is shown in Fig. 1. The quantities concerning the lower half-space H_1 and in the upper half-space H_2 will be denoted by subscript 1 and 2 respectively. We denote the complex rigidity, density and shear velocity in media H_m $(m = 1, 2)$ by M_m, ρ_m and $\beta_m(=\sqrt{M_m/\rho_m})$ respectively. Let the equation of the corrugated interface be given by

$$z = \zeta(x), \tag{1}$$

where ζ is a periodic function of x, independent of y and whose mean value is zero. Fourier series representation of ζ is given by

$$\zeta = \sum_{n=1}^{\infty}\left[\zeta_n e^{\iota nk^*x} + \zeta_{-n} e^{-\iota nk^*x}\right], \tag{2}$$

where ζ_n and ζ_{-n} are Fourier expansion coefficients, k^* is the wave number, n is the series expansion order and $\iota = \sqrt{-1}$. Introducing the constants c, c_n and s_n as follows:

$$\zeta_1 = \zeta_{-1} = \frac{c}{2}, \qquad \zeta_{\pm n} = \frac{c_n \mp \iota s_n}{2}, \quad n = 2, 3, 4, \ldots, \tag{3}$$

we obtain

$$\zeta = c\cos k^*x + \sum_{n=2}^{\infty}[c_n \cos nk^*x + s_n \sin nk^*x].$$

If the interface shape is expressible by only one cosine term, that is, $\zeta = c\cos k^*x$, where c is the amplitude of the corrugation and the wavelength of the corrugation is then given by $2\pi/k^*$.

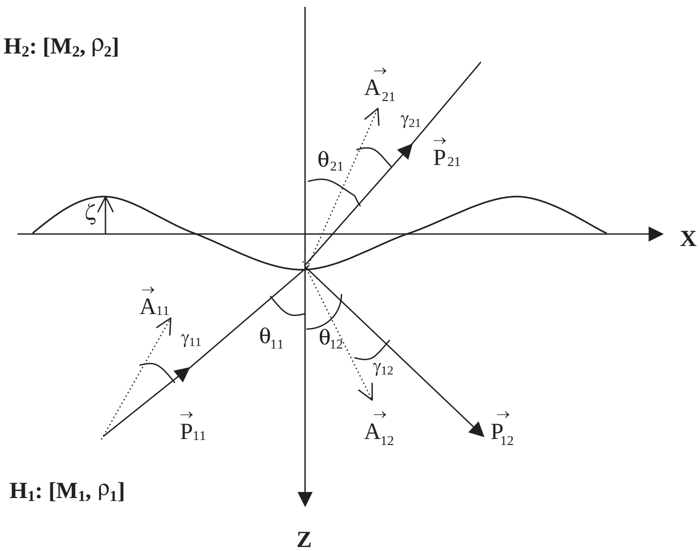

Fig. 1. Geometry of the problem.

The time harmonic equations of small motion for *SH*-wave [i.e. for $\mathbf{U} \propto \exp(\imath\omega t)$] in an isotropic and inhomogeneous visco-elastic media H_m, in the absence of body forces, are given by

$$\frac{\partial}{\partial x}\left(M_m \frac{\partial \mathbf{U}_m}{\partial x}\right) + \frac{\partial}{\partial z}\left(M_m \frac{\partial \mathbf{U}_m}{\partial z}\right) + \rho_m \omega^2 \mathbf{U}_m = 0, \tag{4}$$

where ω is the angular frequency and $\mathbf{U}_m$ is a complex displacement vector given by

$$\mathbf{U}_m = (0, U_{m2}, 0).$$

Since the media are assumed to be laterally and vertically heterogeneous, we take

$$\{M_m, \rho_m\}(x, z) = \{M_{m0}, \rho_{m0}\} \exp(a_m x + b_m z), \tag{5}$$

where ρ_{m0}, a_m, b_m are real constants and M_{m0} are complex constants. Using relation (5) into Eq. (4), the general solution of resulting equation may be written as

$$\mathbf{U}_m = \sum_{j=1}^{2} D_{mj}\left[\exp(-\mathbf{A}_{mj} \cdot \mathbf{r}) \exp\{\imath(\omega t - \mathbf{P}_{mj} \cdot \mathbf{r})\}\right]\hat{y} \quad (m = 1, 2), \tag{6}$$

where $\mathbf{r}$ is the position vector, $\hat{y}$ is the unit base vector and D_{mj} are complex constants. $\mathbf{A}_{mj}$ and $\mathbf{P}_{mj}$ are the attenuation and propagation vectors respectively defined by (Borcherdt, 1977; Kaushik and Chopra, 1984)

$$\mathbf{A}_{mj} = \left[\frac{a_m}{2} - k_{mI}\right]\hat{x} + \left[\frac{b_m}{2} + (-1)^{j+1} d_{\beta_{mI}}\right]\hat{z}, \tag{7}$$

$$\mathbf{P}_{mj} = k_{mR}\hat{x} + (-1)^j d_{\beta_{mR}}\hat{z}, \tag{8}$$

where $\hat{x}$ and $\hat{z}$ are unit base vectors, k_m $(= k_{mR} + \imath k_{mI})$ is the complex horizontal wave number given by (Kaushik and Chopra, 1984)

$$k_m = |\mathbf{P}_{mj}| \sin\theta_{mj} - \imath|\mathbf{A}_{mj}| \sin(\theta_{mj} - \gamma_{mj}), \tag{9}$$

the suffixes R and I refer to the real and imaginary parts respectively and

$$d_{\beta_m}^2 = k_{\beta_m}^2 - k_m^2 - \frac{a_m^2}{4} - \frac{b_m^2}{4}, \quad k_{\beta_m}^2 = \omega^2/\beta_m^2, \quad \beta_m = \sqrt{\frac{M_{m0}}{\rho_{m0}}}, \tag{10}$$

θ_{mj} are defined in Fig. 1 and γ_{mj} are the angles between the propagation and attenuation vectors $\mathbf{P}_{mj}$ and $\mathbf{A}_{mj}$. Using (7) and (8), we see that the attenuation and propagation vectors satisfy the following two relations:

$$|\mathbf{P}_{mj}|^2 - |\mathbf{A}_{mj}|^2 = \mathrm{Re}(k_{\beta_m}^2) - a_m A_{mjx} - b_m A_{mjz}, \tag{11}$$

$$2|\mathbf{P}_{mj}||\mathbf{A}_{mj}| \cos\gamma_{mj} = -\mathrm{Im}(k_{\beta_m}^2) + a_m P_{mjx} + b_m P_{mjz}, \tag{12}$$

where A_{mjx}, A_{mjz}, P_{mjx} and P_{mjz} are the components of attenuation and propagation vectors in the directions shown by their suffixes x and z; Re stands for real part and Im stands for imaginary part. On solving Eqs. (11) and (12), one can obtain

$$2|\mathbf{P}_{mj}|^2 = \Omega_{m1} + \sqrt{\Omega_{m1}^2 + \Omega_{m2}^2}, \tag{13}$$

$$2|\mathbf{A}_{mj}|^2 = -\Omega_{m1} + \sqrt{\Omega_{m1}^2 + \Omega_{m2}^2}, \tag{14}$$

where

$$\Omega_{m1} = \mathrm{Re}(k_{\beta_m}^2) - a_m A_{mjx} - b_m A_{mjz}, \tag{15}$$

$$\Omega_{m2} = [-\mathrm{Im}(k_{\beta_m}^2) + a_m P_{mjx} + b_m P_{mjz}]/\cos\gamma_{mj}. \tag{16}$$

Let a train of general plane SH-wave propagating through the lower half space H_1 become incident at the corrugated interface $z = \zeta$ and making an angle θ_{11} with the z-axis. Due to corrugation of the interface, the reflection and refraction phenomena will be effected and the incident SH-wave will give rise to the following waves at the corrugated interface:

In medium H_1 (i) a regularly reflected wave making an angle θ_{12} with z-axis, (ii) a spectrum of nth order of irregularly reflected waves at angles θ_{12}^n in the left side of regularly reflected wave and (iii) a similar spectrum of irregularly reflected waves at angles $\theta_{12}'^n$ in the right side of regularly reflected wave.

In medium H_2 (i) a regularly refracted wave at an angle θ_{21} with z-axis, (ii) a spectrum of nth order of irregularly refracted waves at angles θ_{21}^n in the left side of regularly refracted wave and (iii) a similar spectrum of irregularly refracted waves at angles $\theta_{21}'^n$ in the right side of regularly refracted wave.

Thus, the total displacement field in medium H_1 will be the sum of incident, regularly reflected and irregularly reflected waves, given as

$$u_{12} = \sum_{j=1}^{2} D_{1j} \exp\{-\mathbf{A}_{1j}\cdot\mathbf{r}\}\exp\{\imath(\omega t - \mathbf{P}_{1j}\cdot\mathbf{r})\} + \sum_{n=1}^{\infty} B_n \exp\{-\mathbf{A}_{12}^n\cdot\mathbf{r}\}\exp\{\imath(\omega t - \mathbf{P}_{12}^n\cdot\mathbf{r})\}$$
$$+ \sum_{n=1}^{\infty} B_n' \exp\{-\mathbf{A}_{12}'^n\cdot\mathbf{r}\}\exp\{\imath(\omega t - \mathbf{P}_{12}'^n\cdot\mathbf{r})\}. \tag{17}$$

Similarly, the total displacement field in medium H_2 will be given by

$$u_{22} = D_{21} \exp\{-\mathbf{A}_{21}\cdot\mathbf{r}\}\exp\{\imath(\omega t - \mathbf{P}_{21}\cdot\mathbf{r})\} + \sum_{n=1}^{\infty} C_n \exp\{-\mathbf{A}_{21}^n\cdot\mathbf{r}\}\exp\{\imath(\omega t - \mathbf{P}_{21}^n\cdot\mathbf{r})\}$$
$$+ \sum_{n=1}^{\infty} C_n' \exp\{-\mathbf{A}_{21}'^n\cdot\mathbf{r}\}\exp\{\imath(\omega t - \mathbf{P}_{21}'^n\cdot\mathbf{r})\}, \tag{18}$$

where D_{11}, D_{12} and D_{21} are in general frequency dependent complex amplitudes of the incident, reflected and transmitted waves respectively; B_n and B_n' are the amplitudes of irregularly reflected (scattered) waves; C_n and C_n' are the amplitudes of irregularly refracted waves and

$$\mathbf{A}_{mj}^n = \left[\frac{a_m}{2} - k_{mI}^n\right]\hat{x} + \left[\frac{b_m}{2} + (-1)^{j+1} d_{\beta_{mI}}^n\right]\hat{z}, \quad \mathbf{P}_{mj}^n = k_{mR}^n\hat{x} + (-1)^j d_{\beta_{mR}}^n\hat{z}, \tag{19}$$

$$\mathbf{A}_{mj}'^n = \left[\frac{a_m}{2} - k_{mI}'^n\right]\hat{x} + \left[\frac{b_m}{2} + (-1)^{j+1} d_{\beta_{mI}}'^n\right]\hat{z}, \quad \mathbf{P}_{mj}'^n = k_{mR}'^n\hat{x} + (-1)^j d_{\beta_{mR}}'^n\hat{z}, \tag{20}$$

k_m^n and $k_m'^n$ are the complex horizontal wave numbers for the waves of nth order spectrum in H_m and are given by

$$k_m^n = |\mathbf{P}_{mj}^n|\sin\theta_{mj}^n - \imath|\mathbf{A}_{mj}^n|\sin(\theta_{mj}^n - \gamma_{mj}^n) \quad (m \neq j), \tag{21}$$

$$k_m'^n = |\mathbf{P}_{mj}'^n|\sin\theta_{mj}'^n - \imath|\mathbf{A}_{mj}'^n|\sin(\theta_{mj}'^n - \gamma_{mj}'^n) \quad (m \neq j), \tag{22}$$

and

$$(d_{\beta_m}^n)^2 = k_{\beta_m}^2 - (k_m^n)^2 - \frac{1}{4}(a_m^2 + b_m^2), \quad (d_{\beta_m}'^n)^2 = k_{\beta_m}^2 - (k_m'^n)^2 - \frac{1}{4}(a_m^2 + b_m^2). \tag{23}$$

3. Boundary conditions

The boundary conditions to be satisfied at the corrugated interface are the continuity of displacement and traction, that is, at $z=\zeta$

$$u_{12}=u_{22}, \tag{24}$$

$$M_{10}\left[\frac{\partial u_{12}}{\partial z}-\frac{\partial u_{12}}{\partial x}\zeta'\right]=M_{20}\left[\frac{\partial u_{22}}{\partial z}-\frac{\partial u_{22}}{\partial x}\zeta'\right], \tag{25}$$

where ζ' is the derivative of ζ with respect to x. Inserting the values of u_{12} and u_{22} given by Eqs. (17) and (18) into the above boundary conditions, we have

$$\begin{aligned}
&D_{11}\exp\left\{-\left(\frac{a_1}{2}+\imath k_1\right)x-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)\zeta\right\}+D_{12}\exp\left\{-\left(\frac{a_1}{2}+\imath k_1\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)\zeta\right\}\\
&\quad+\sum_{n=1}^{\infty}B_n\exp\left\{-\left(\frac{a_1}{2}+\imath k_1^n\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\zeta\right\}+\sum_{n=1}^{\infty}B'_n\exp\left\{-\left(\frac{a_1}{2}+\imath k_1'^n\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\zeta\right\}\\
&=D_{21}\exp\left\{-\left(\frac{a_2}{2}+\imath k_2\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\zeta\right\}+\sum_{n=1}^{\infty}C_n\exp\left\{-\left(\frac{a_2}{2}+\imath k_2^n\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\zeta\right\}\\
&\quad+\sum_{n=1}^{\infty}C'_n\exp\left\{-\left(\frac{a_2}{2}+\imath k_2'^n\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\zeta\right\},
\end{aligned} \tag{26}$$

and

$$\begin{aligned}
&M_{10}\Bigg[\left(-\frac{b_1}{2}+\imath d_{\beta_1}\right)D_{11}\exp\left\{-\left(\frac{a_1}{2}+\imath k_1\right)x-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)\zeta\right\}-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)D_{12}\exp\\
&\quad\times\left\{-\left(\frac{a_1}{2}+\imath k_1\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)\zeta\right\}-\sum_{n=1}^{\infty}B_n\left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\exp\left\{-\left(\frac{a_1}{2}+\imath k_1^n\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\zeta\right\}\\
&\quad-\sum_{n=1}^{\infty}B'_n\left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\exp\left\{-\left(\frac{a_1}{2}+\imath k_1'^n\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\zeta\right\}\\
&\quad+\zeta'\Bigg[\left(\frac{a_1}{2}+\imath k_1\right)D_{11}\exp\left\{-\left(\frac{a_1}{2}+\imath k_1\right)x-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)\zeta\right\}+\left(\frac{a_1}{2}+\imath k_1\right)D_{12}\exp\\
&\quad\times\left\{-\left(\frac{a_1}{2}+\imath k_1\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)\zeta\right\}+\sum_{n=1}^{\infty}B_n\left(\frac{a_1}{2}+\imath k_1^n\right)\exp\left\{-\left(\frac{a_1}{2}+\imath k_1^n\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\zeta\right\}\\
&\quad+\sum_{n=1}^{\infty}B'_n\left(\frac{a_1}{2}+\imath k_1'^n\right)\exp\left\{\left(-\frac{a_1}{2}+\imath k_1'^n\right)x-\left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\zeta\right\}\Bigg]\Bigg]\\
&=M_{20}\Bigg[\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)D_{21}\exp\left\{-\left(\frac{a_2}{2}+\imath k_2\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\zeta\right\}-\sum_{n=1}^{\infty}C_n\left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\exp\\
&\quad\times\left\{-\left(\frac{a_2}{2}+\imath k_2^n\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\zeta\right\}-\sum_{n=1}^{\infty}C'_n\left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\exp\left\{-\left(\frac{a_2}{2}+\imath k_2'^n\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\zeta\right\}\\
&\quad+\zeta'\Bigg[\left(\frac{a_2}{2}+\imath k_2\right)D_{21}\exp\left\{-\left(\frac{a_2}{2}+\imath k_2\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\zeta\right\}+\sum_{n=1}^{\infty}C_n\left(\frac{a_2}{2}+\imath k_2^n\right)\exp\\
&\quad\times\left\{-\left(\frac{a_1}{2}+\imath k_2^n\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\zeta\right\}+\sum_{n=1}^{\infty}C'_n\left(\frac{a_2}{2}+\imath k_2'^n\right)\exp\left\{-\left(\frac{a_2}{2}+\imath k_2'^n\right)x-\left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\zeta\right\}\Bigg]\Bigg].
\end{aligned} \tag{27}$$

Of course, Eqs. (26) and (27) imply the Snell's law, which is not affected by the inhomogeneity of the medium and, according to Borcherdt (1977) and the representation (9) can be written as

$$k = |\mathbf{P}|\sin\theta - \imath|\mathbf{A}|\sin(\theta-\gamma) = k_1 = k_2, \tag{28}$$

where $|\mathbf{P}|$ and $|\mathbf{A}|$ are the values of $|\mathbf{P}_{11}|$ and $|\mathbf{A}_{11}|$ given by (13) and (14) with $a_1 = b_1 = 0$ respectively.

Eq. (28) and the fact that the amplitudes D_{11}, D_{12}, D_{21}, B_n, B'_n, C_n and C'_n are independent of the spatial coordinates imply $a_1 = a_2 = a$ (say). In addition, owing to Eq. (2), we obtain the extended form of concerned Spectrum theorem (Asano, 1960) which gives

$$k_i^n - k_i = nk^*, \quad k_i'^n - k_i = -nk^* \qquad (i = 1, 2) \tag{29}$$

Using Eqs. (21), (22) and (28), it can be verified that when visco-elasticity is absent, the Spectrum theorem given by (29) reduces to that given in Asano (1960). Now using Eqs. (28) and (29) into Eqs. (26) and (27), we obtain

$$\begin{aligned}
&D_{11}\exp\left\{-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)\zeta\right\} + D_{12}\exp\left\{-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)\zeta\right\} + \sum_{n=1}^{\infty} B_n\exp\left\{-\frac{b_1}{2}\zeta - \imath(nk^*x + d_{\beta_1}^n\zeta)\right\}\\
&\quad + \sum_{n=1}^{\infty} B'_n\exp\left\{-\frac{b_1}{2}\zeta + \imath(nk^*x - d_{\beta_1}'^n\zeta)\right\}\\
&= D_{21}\exp\left\{-\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\zeta\right\} + \sum_{n=1}^{\infty} C_n\exp\left\{-\frac{b_2}{2}\zeta - \imath(nk^*x - d_{\beta_2}^n\zeta)\right\}\\
&\quad + \sum_{n=1}^{\infty} C'_n\exp\left\{-\frac{b_2}{2}\zeta + \imath(nk^*x + d_{\beta_2}'^n\zeta)\right\},
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
&M_{10}\Bigg[\left(-\frac{b_1}{2}+\imath d_{\beta_1}\right)D_{11}\exp\left\{-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)\zeta\right\} - \left(\frac{b_1}{2}+\imath d_{\beta_1}\right)D_{12}\exp\left\{-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)\zeta\right\}\\
&\quad -\sum_{n=1}^{\infty} B_n\left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\exp\left\{-\imath nk^*x - \left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\zeta\right\} - \sum_{n=1}^{\infty} B'_n\left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\\
&\quad \times\exp\left\{\imath nk^*x - \left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\zeta\right\} + \zeta'\Bigg[\frac{a_1}{2}D_{11}\exp\left\{-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)\zeta\right\} + \frac{a_1}{2}D_{12}\\
&\quad \times\exp\left\{-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)\zeta\right\} + \sum_{n=1}^{\infty} B_n\left(\frac{a_1}{2}+\imath nk^*\right)\exp\left\{-\imath nk^*x - \left(\frac{b_1}{2}+\imath d_{\beta_1}^n\right)\zeta\right\}\\
&\quad + \sum_{n=1}^{\infty} B'_n\left(\frac{a_1}{2}-\imath nk^*\right)\exp\left\{\imath nk^*x - \left(\frac{b_1}{2}+\imath d_{\beta_1}'^n\right)\zeta\right\}\Bigg]\Bigg]\\
&= M_{20}\Bigg[\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)D_{21}\exp\left\{-\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\zeta\right\} - \sum_{n=1}^{\infty} C_n\left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\exp\left\{-\imath nk^*x - \left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\zeta\right\}\\
&\quad -\sum_{n=1}^{\infty} C'_n\left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\exp\left\{\imath nk^*x - \left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\zeta\right\} + \zeta'\Bigg[\frac{a_2}{2}D_{21}\exp\left\{-\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\zeta\right\}\\
&\quad + \sum_{n=1}^{\infty} C_n\left(\frac{a_2}{2}+\imath nk^*\right)\exp\left\{-\imath nk^*x - \left(\frac{b_2}{2}-\imath d_{\beta_2}^n\right)\zeta\right\} + \sum_{n=1}^{\infty} C'_n\left(\frac{a_2}{2}-\imath nk^*\right)\exp\left\{\imath nk^*x - \left(\frac{b_2}{2}-\imath d_{\beta_2}'^n\right)\zeta\right\}\Bigg]\Bigg].
\end{aligned} \tag{31}$$

Eqs. (30) and (31) enable us to determine the reflection and transmission coefficients of any order of approximation of the corrugated interface.

4. Solution of the first order approximation

We assume that the corrugation of the interface $z = \zeta(x)$ is so small that higher powers of ζ can be neglected, so we have

$$e^{-\imath Q\zeta} = 1 - \imath Q\zeta. \tag{32}$$

Now, collecting the terms independent of x and ζ to both side of Eqs. (30) and (31), we obtain

$$D_{11} + D_{12} = D_{21}, \tag{33}$$

$$M_{10}\left[\left(\frac{b_1}{2} - \imath d_{\beta_1}\right)D_{11} + \left(\frac{b_1}{2} + \imath d_{\beta_1}\right)D_{12}\right] = M_{20}\left(\frac{b_2}{2} - \imath d_{\beta_2}\right)D_{21}. \tag{34}$$

Using the notations $R = \frac{D_{12}}{D_{11}}$ and $T = \frac{D_{21}}{D_{11}}$, the solution of Eqs. (33) and (34) is given by

$$R = \frac{\frac{1}{2}\left(b_2 - b_1\frac{M_{10}}{M_{20}}\right) + \imath\left(\frac{M_{10}}{M_{20}}d_{\beta_1} - d_{\beta_2}\right)}{\frac{1}{2}\left(-b_2 + b_1\frac{M_{10}}{M_{20}}\right) + \imath\left(\frac{M_{10}}{M_{20}}d_{\beta_1} + d_{\beta_2}\right)}, \tag{35}$$

$$T = \frac{2\imath\frac{M_{10}}{M_{20}}d_{\beta_1}}{\frac{1}{2}\left(-b_2 + b_1\frac{M_{10}}{M_{20}}\right) + \imath\left(\frac{M_{10}}{M_{20}}d_{\beta_1} + d_{\beta_2}\right)}. \tag{36}$$

These are the reflection and refraction coefficients for the plane interface between two laterally and vertically heterogeneous viscoelastic solids already obtained by Kaushik and Chopra (1984) for the relevant problem. Further, to find the values for the solution of first order approximation for B_n and C_n, we collect the coefficients of $e^{-\imath nk^*x}$ to both sides of Eqs. (30) and (31), obtaining

$$R_n - T_n = \left[(1+R)\frac{b_1}{2} - \imath(1-R)d_{\beta_1} + T\left(-\frac{b_2}{2} + \imath d_{\beta_2}\right)\right]\zeta_{-n}, \tag{37}$$

$$\begin{aligned}\left(\frac{b_1}{2} + \imath d^n_{\beta_1}\right)R_n - \frac{M_{20}}{M_{10}}\left(\frac{b_2}{2} - \imath d^n_{\beta_2}\right)T_n = \Bigg[\left(-\frac{b_1}{2} + \imath d_{\beta_1}\right)^2 - \frac{\imath nk^*a}{2}(1+R) \\ + \left(\frac{b_1}{2} + \imath d_{\beta_1}\right)^2 R - \frac{M_{20}}{M_{10}}T\left[\left(-\frac{b_2}{2} + \imath d_{\beta_2}\right)^2 - \frac{\imath nk^*a}{2}\right]\Bigg]\zeta_{-n},\end{aligned} \tag{38}$$

where $R_n = \frac{B_n}{D_{11}}$ and $T_n = \frac{C_n}{D_{11}}$.

Similarly, on equating the coefficients of $e^{\imath nk^*x}$ to both sides of Eqs. (30) and (31), we obtain the first order approximation for B'_n and C'_n as follows:

$$R'_n - T'_n = \left[(1+R)\frac{b_1}{2} - \imath(1-R)d_{\beta_1} + T\left(-\frac{b_2}{2} + \imath d_{\beta_2}\right)\right]\zeta_n, \tag{39}$$

$$\left(\frac{b_1}{2}+\imath d'^{n}_{\beta_1}\right)R'_n-\frac{M_{20}}{M_{10}}\left(\frac{b_2}{2}-\imath d'^{n}_{\beta_2}\right)T'_n$$
$$=\left[\left(-\frac{b_1}{2}+\imath d_{\beta_1}\right)^2+\frac{\imath nk^*a}{2}(1+R)+\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2R-\frac{M_{20}}{M_{10}}T\left[(-\frac{b_2}{2}+\imath d_{\beta_2})^2+\frac{\imath nk^*a}{2}\right]\right]\zeta_n, \tag{40}$$

where $R'_n=\frac{B'_n}{D_{11}}$ and $T'_n=\frac{C'_n}{D_{11}}$.

Eqs. (37)–(40) give the values of R_n, T_n, R'_n and T'_n as follows:

$$R_n=\frac{\Delta_{R_n}}{\Delta_n},\quad T_n=\frac{\Delta_{T_n}}{\Delta_n},\quad R'_n=\frac{\Delta_{R'_n}}{\Delta'_n},\quad T'_n=\frac{\Delta_{T'_n}}{\Delta'_n}, \tag{41}$$

where

$$\Delta_{R_n}=\left[\frac{M_{20}}{M_{10}}\left(\frac{b_2}{2}-\imath d^{n}_{\beta_2}\right)\right]\left[\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)+R\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)+T\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)\right]$$
$$+\frac{M_{20}}{M_{10}}T\left[\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)^2-\frac{\imath nk^*a}{2}\right]-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2+\frac{\imath nk^*a}{2}-R\left[\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2-\frac{\imath nk^*a}{2}\right]\Bigg]\zeta_{-n}, \tag{42}$$

$$\Delta_{T_n}=-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2+\frac{\imath nk^*a}{2}+R\left[-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2+\frac{\imath nk^*a}{2}+\frac{M_{20}}{M_{10}}T\left[\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)^2-\frac{\imath nk^*a}{2}\right]\right.$$
$$\left.+\left(\frac{b_1}{2}+\imath d^{n}_{\beta_1}\right)\left[(1+R)\frac{b_1}{2}-(1-R)\imath d_{\beta_1}-T\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\right]\right]\zeta_{-n}, \tag{43}$$

$$\Delta_{R'_n}=\left[\frac{M_{20}}{M_{10}}\left(\frac{b_2}{2}-\imath d'^{n}_{\beta_2}\right)\right]\left[\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)+R\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)+T\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)\right]$$
$$+\frac{M_{20}}{M_{10}}T\times\left[\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)^2+\frac{\imath nk^*a}{2}\right]-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2-\frac{\imath nk^*a}{2}+R\left[\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2-\frac{\imath nk^*a}{2}\right]\Bigg]\zeta_n, \tag{44}$$

$$\Delta_{T'_n}=\left[-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2-\frac{\imath nk^*a}{2}-R\left[\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2+\frac{\imath nk^*a}{2}\right]+\frac{M_{20}}{M_{10}}T\left[\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)^2+\frac{\imath nk^*a}{2}\right]\right.$$
$$\left.+\left(\frac{b_1}{2}+\imath d'^{n}_{\beta_1}\right)\left[(1+R)\frac{b_1}{2}-(1-R)\imath d_{\beta_1}+T\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)\right]\right]\zeta_n, \tag{45}$$

$$\Delta_n=\left[-\frac{b_1}{2}+\frac{b_2M_{20}}{2M_{10}}-\imath\left(d^{n}_{\beta_1}+\frac{M_{20}}{M_{10}}d^{n}_{\beta_2}\right)\right], \tag{46}$$

$$\Delta'_n=\left[-\frac{b_1}{2}+\frac{b_2M_{20}}{2M_{10}}-\imath\left(d'^{n}_{\beta_1}+\frac{M_{20}}{M_{10}}d'^{n}_{\beta_2}\right)\right]. \tag{47}$$

The values of the coefficients R and T appearing in the above expressions are given by Eqs. (35) and (36). Here R_n and R'_n are reflection coefficients for the first order approximation of the corrugation; T_n and T'_n are refraction coefficients for the first order approximation.

In a special case, if we consider $\zeta_n=\zeta_{-n}=0$; $(n\neq 1)$ and $\zeta_1=\zeta_{-1}=\frac{c}{2}$ then the boundary surface is given by $z=c\cos k^*x$, where c is the amplitude of the corrugation. From Eqs. (42)–(47) we obtain the following formulae of R_1, T_1, R'_1 and T'_1 for the first order approximation of the corrugation as

$$R_1=\frac{\Delta_{R_1}}{\Delta_1},\quad T_1=\frac{\Delta_{T_1}}{\Delta_1},\quad R_1'=\frac{\Delta_{R_1'}}{\Delta_1'},\quad T_1'=\frac{\Delta_{T_1'}}{\Delta_1'},\tag{48}$$

where $\Delta_{R_1},\Delta_{T_1},\Delta_{R_1'}$, $\Delta_{T_1'},\Delta_1$ and Δ_1' are given as

$$\begin{aligned}\Delta_{R_1}=\frac{c}{2}\Bigg[&\frac{M_{20}}{M_{10}}\left(\frac{b_2}{2}-\imath d^1_{\beta_2}\right)\left[\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)+R\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)+T\left(-\frac{b_2}{2}+\imath d_{\beta_2}\right)\right]\\&+\frac{M_{20}}{M_{10}}T\times\left[\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)^2-\frac{\imath k^*a}{2}\right]-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2+\frac{\imath k^*a}{2}+R\left[-\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2+\frac{\imath k^*a}{2}\right]\Bigg],\end{aligned}\tag{49}$$

$$\begin{aligned}\Delta_{T_1}=\frac{c}{2}\Bigg[&-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2+\frac{\imath k^*a}{2}-R\left[\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2-\frac{\imath k^*a}{2}\right]+\frac{M_{20}}{M_{10}}T\left[\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)^2-\frac{\imath k^*a}{2}\right]\\&+\left(\frac{b_1}{2}+\imath d^1_{\beta_1}\right)\left[(1+R)\frac{b_1}{2}-(1-R)\imath d_{\beta_1}-T\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\right]\Bigg],\end{aligned}\tag{50}$$

$$\begin{aligned}\Delta_{R_1'}=\frac{c}{2}\Bigg[&\frac{M_{20}}{M_{10}}\left(\frac{b_2}{2}-\imath d'^1_{\beta_2}\right)\left[\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)+R\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)-T\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\right]\\&+\frac{M_{20}}{M_{10}}T\times\left[\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)^2+\frac{\imath k^*a}{2}\right]-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2-\frac{\imath k^*a}{2}+R\left[\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2-\frac{\imath k^*a}{2}\right]\Bigg],\end{aligned}\tag{51}$$

$$\begin{aligned}\Delta_{T_1'}=\frac{c}{2}\Bigg[&-\left(\frac{b_1}{2}-\imath d_{\beta_1}\right)^2-\frac{\imath k^*a}{2}-R\left[\left(\frac{b_1}{2}+\imath d_{\beta_1}\right)^2+\frac{\imath k^*a}{2}\right]+\frac{M_{20}}{M_{10}}T\left[\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)^2+\frac{\imath k^*a}{2}\right]\\&+\left(\frac{b_1}{2}+\imath d'^1_{\beta_1}\right)\left[(1+R)\frac{b_1}{2}-(1-R)\imath d_{\beta_1}-T\left(\frac{b_2}{2}-\imath d_{\beta_2}\right)\right]\Bigg],\end{aligned}\tag{52}$$

$$\Delta_1=\frac{1}{2}\left(\frac{M_{20}}{M_{10}}b_2-b_1\right)-\imath\left(d^1_{\beta_1}+\frac{M_{20}}{M_{10}}d^1_{\beta_2}\right),$$

$$\Delta_1'=\frac{1}{2}\left(\frac{M_{20}}{M_{10}}b_2-b_1\right)-\imath\left(d'^1_{\beta_1}+\frac{M_{20}}{M_{10}}d'^1_{\beta_2}\right).$$

The values of d_{β_m}, $d^1_{\beta_m}$ and $d'^1_{\beta_m}$ can be obtained from Eqs. (10) and (23) by taking $n=1$ and $a_1=a_2=a$. We notice from the above equations that the coefficients for first order of approximation are proportional to amplitude of the corrugated interface and are functions of physical properties of the media and of the incident wave.

5. Particular cases

(a) When only lateral heterogeneity of both the media is removed, we are left with vertically heterogeneous viscoelastic media. In this case, we shall have $a_1=a_2=a=0$. Plugging these values into Eqs. (10) and (23), we get

$$d^2_{\beta_m}=k^2_{\beta_m}-k^2_m-\frac{b^2_m}{4},\quad (d^n_{\beta_m})^2=k^2_{\beta_m}-(k^n_m)^2-\frac{b^2_m}{4},\quad (d'^n_{\beta_m})^2=k^2_{\beta_m}-(k'^n_m)^2-\frac{b^2_m}{4}.$$

With these modified values, the reflection and transmission coefficients due to incident SH-waves at a corrugated interface between two different vertically heterogeneous viscoelastic solids can be obtained from Eqs. (35), (36) and (41).

(b) When only vertical heterogeneity of both the media is removed, we shall have laterally heterogeneous viscoelastic media. In this case, we shall have $b_1 = b_2 = 0$, so that the expressions in Eqs. (10) and (23) reduce to

$$d^2_{\beta_m} = k^2_{\beta_m} - k^2_m - \frac{a^2_m}{4}, \quad (d^n_{\beta_m})^2 = k^2_{\beta_m} - (k^n_m)^2 - \frac{a^2_m}{4}, \quad (d'^n_{\beta_m})^2 = k^2_{\beta_m} - (k'^n_m)^2 - \frac{a^2_m}{4}.$$

With these values, the reflection and transmission coefficients, in this case, can be obtained from Eqs. (35), (36) and (41).

(c) When only viscosity of the media is removed, we are left with isotropic and heterogeneous elastic media. Replacing T, T_n and T'_n by $\sqrt{M_{10}/M_{20}}T_0$, $\sqrt{M_{10}/M_{20}}T_n$ and $\sqrt{M_{10}/M_{20}}T'_n$ respectively; $d^2_{\beta_1} = k^2_{\beta_1} - k^2_1 - \frac{a^2}{4} - \frac{b^2_1}{4} = s^2, d^2_{\beta_2} = k^2_{\beta_2} - k^2_2 - \frac{a^2}{4} - \frac{b^2_2}{4} = r^2$. It is easy to see that a^2_m and b^2_m reduce to $4a^2_0$ and $4b^2_m$ in the notations of Gupta (1987). With these values and $b_m = \gamma_m$, Eqs. (35)–(40) match with Eqs. (28), (29) and (32)–(35) of Gupta (1987) for the corresponding problem.

(d) When viscosity, lateral and vertical heterogeneities of the media are removed, then the medium H_1 and H_2 becomes uniform elastic. In this case, putting $a_m = b_m = 0$ into the expressions of Eq. (10), we have $d_{\beta_1} = \frac{\omega\cos\theta_{11}}{\beta_{h_1}}$ and $d_{\beta_2} = \frac{\omega\cos\theta_{21}}{\beta_{h_2}}$, where $\beta_{h_1} = \sqrt{M_{10}/\rho_{10}}, \beta_{h_2} = \sqrt{M_{20}/\rho_{20}}$. With the help of these reductions and using appropriate notations, the reflection and transmission coefficients for the plane interface given by Eqs. (35) and (36) and the boundary conditions (42)–(47) yielding reflection and transmission coefficients for the first order approximation, match with those of Asano (1960) for the relevant case.

(e) When corrugation, viscosity, lateral and vertical heterogeneities of the media are removed, the problem reduces to the problem of SH-wave incident at plane interface between two homogeneous elastic half spaces. All the reflection and transmission coefficients R_1, T_1, R'_1 and T'_1 for the first order approximation of the corrugation, in this case, vanish as they are proportional to the amplitude of corrugation c. Therefore, in this case each of c, a, b_1 and b_2 vanishes. Thus from Eq. (48), we see that the coefficients R_1, T_1, R'_1 and T'_1 become zero and the expressions of R and T given by (35) and (36) reduce to those given in Savarensky (1975) for the relevant problem, after replacing M_{10}/M_{20} by m and β_{h_1}/β_{h_2} by n.

(f) When corrugation and viscosity in the media are removed, we are left with the reflection and transmission of SH-waves at the plane boundary between two laterally and vertically heterogeneous solids. In this case, $\zeta = 0, M_{10}/M_{20} = m$, $a_1 = a_2 = \alpha$, $b_1 = \gamma_1$, and $b_2 = \gamma_2$ and the expressions in Eq. (10) take the form $d^2_{\beta_m} = k^2_{\beta_m}\cos^2\theta_m - \frac{\alpha^2_m}{4} - \frac{\gamma^2_m}{4} = s^2_m$. With these modifications, it can be seen that the reflection and transmission coefficients given by Eqs. (35) and (36) at the plane interface are in full agreement with those given in Singh et al. (1978) for the relevant problem.

(g) When lateral and vertical heterogeneities from both the media and visco-elasticity from the upper medium is removed, the problem reduces to the problem of reflection and transmission of SH-waves at a corrugated interface between elastic and homogeneous viscoelastic solid half spaces. In this case, $a = b_1 = b_2 = 0$ and $k_{2I} = d_{\beta_{2I}} = 0$. From Eq. (7) $A_{mj} = 0$, which means $\gamma_{mj} = 0$. Thus the expressions in Eq. (10) reduce to $d^2_{\beta_m} = k^2_{\beta_m} - k^2_m$. The reduced formulae of reflection and transmission coefficients from Eqs. (35), (36) and (41) at plane and corrugated interface become

$$R = \frac{M_{10}d_{\beta_1} - M_{20}d_{\beta_2}}{M_{10}d_{\beta_1} + M_{20}d_{\beta_2}}, \quad T = \frac{2M_{10}d_{\beta_1}}{M_{10}d_{\beta_1} + M_{20}d_{\beta_2}},$$

$$R_1 = \frac{\iota c}{2(M_{10}d^1_{\beta_1} + M_{20}d^1_{\beta_2})}\left[M_{20}\{(1-R)d^1_{\beta_2}d_{\beta_1} + Td_{\beta_2}(d_{\beta_2} - d^1_{\beta_2})\} - M_{10}(1+R)d^2_{\beta_1}\right],$$

$$T_1 = \frac{-\iota c}{2(M_{10}d^1_{\beta_1} + M_{20}d^1_{\beta_2})}\left[M_{10}\{(1+R)d^2_{\beta_1} + (1-R)d_{\beta_1}d^1_{\beta_1}\} - T(M_{20}d^2_{\beta_2} + M_{10}d^1_{\beta_1}d_{\beta_2})\right],$$

$$R'_1 = \frac{\iota c}{2(M_{10}d'^1_{\beta_1} + M_{20}d'^1_{\beta_2})}\left[M_{20}\{(1-R)d'^1_{\beta_2}d_{\beta_1} - Td_{\beta_2}(d_{\beta_2} + d'^1_{\beta_2})\} + M_{10}(1+R)d^2_{\beta_1}\right],$$

$$T'_1 = \frac{-\iota c}{2(M_{10}d'^1_{\beta_1} + M_{20}d'^1_{\beta_2})}\left[M_{10}\{(1+R)d^2_{\beta_1} + (1-R)d_{\beta_1}d'^1_{\beta_1}\} - T(M_{20}d^2_{\beta_2} + M_{10}d'^1_{\beta_1}d_{\beta_2})\right].$$

(h) When lateral and vertical heterogeneity in the upper medium is removed and only lateral heterogeneity in the lower medium is removed then we shall be left with the problem of reflection and refraction of *SH* waves at a corrugated interface between homogeneous and vertically inhomogeneous viscoelastic half-spaces. In this case, the corresponding formulae for reflection and transmission coefficients at the corrugated interface can be obtained easily by putting $a = b_1 = 0$. It is easy to verify that if corrugation is removed in addition to the above then the formulae (36) and (37) match with the formulae (35) and (36) obtained by Kaushik and Chopra (1980) for the relevant problem.

6. Numerical results and discussion

In order to study the effect of inhomogeneities (lateral and vertical), corrugation and the angle between attenuation and propagation vectors numerically on reflection and transmission coefficients, when a plane *SH*-wave become incident obliquely at a corrugated interface between two viscoelastic half spaces H_1 and H_2, we have computed the modulus values of these coefficients for a specific model. We take the following values of relevant elastic parameters as given in Silva (1976) and Krebes and Hron (1980a,b):

$$M_{10R}/M_{20R} = 2.64, \quad M_{10I}/M_{10R} = 45.0, \quad \rho_1 = 2.2\ \text{g/cm}^3$$

$$M_{20I}/M_{20R} = 30.0, \quad \beta_{1R}/\beta_{2R} = 1.46, \quad \rho_2 = 3.3\ \text{g/cm}^3$$

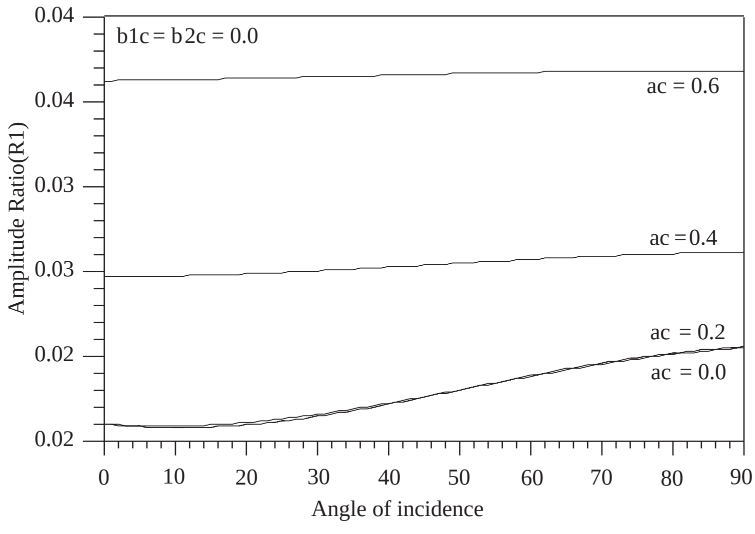

Fig. 2. Variation of R_1 with θ when $b_1c = 0 = b_2c$ and $ac = 0.0, 0.2, 0.4, 0.6$.

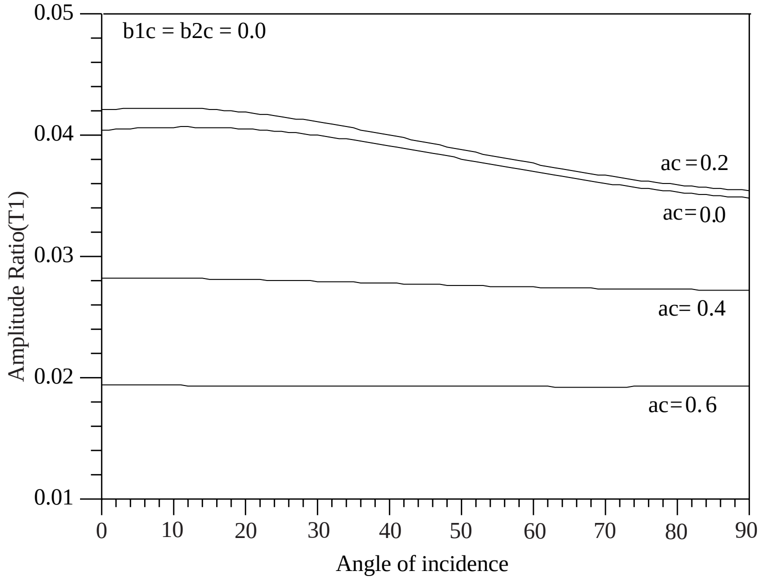

Fig. 3. Variation of T_1 with θ when $b_1c = 0 = b_2c$ and $ac = 0.0$, 0.2, 0.4, 0.6.

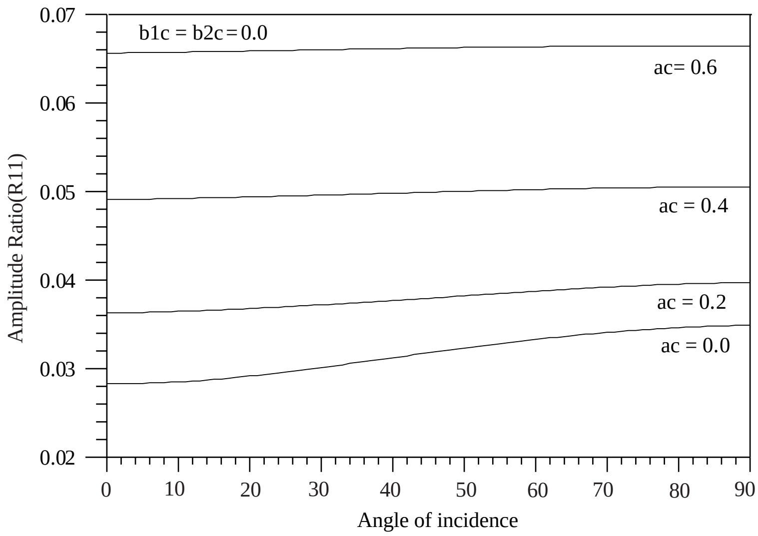

Fig. 4. Variation of R_1' with θ when $b_1c = 0 = b_2c$ and $a\ c = 0.0$, 0.2, 0.4, 0.6.

$\omega c/\beta_{h_1} = 0.5$, $\gamma_{11} = 20°$, $\theta_{11} = 45°$ and k^*c=0.00125, wherever not mentioned. Hereafter, we shall use θ and γ instead of θ_{11} and γ_{11} respectively. The non-dimensional quantities b_1c and b_2c represent the vertical heterogeneity factors in H_1 and H_2 respectively, while ac is lateral heterogeneity factor in both the media. For the elastic model, we shall take the values of parameters, $M_{10I} = M_{20I}$=0.

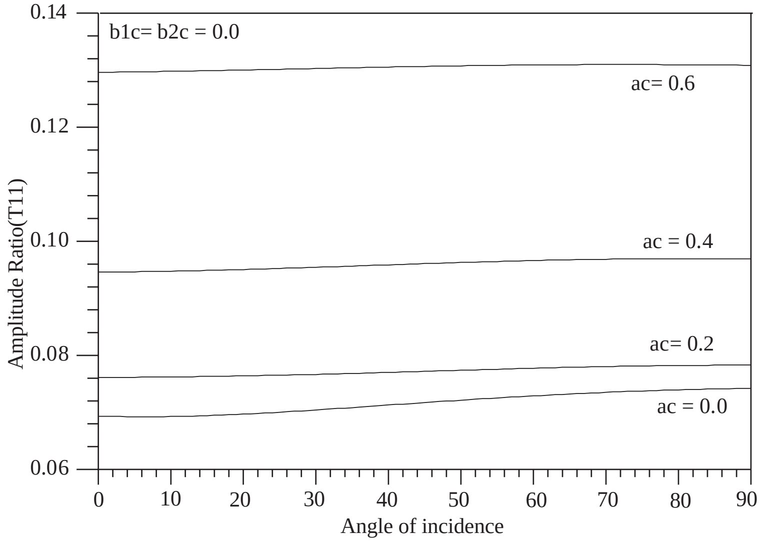

Fig. 5. Variation of T_1' with θ when $b_1c = 0 = b_2c$ and $ac = 0.0, 0.2, 0.4, 0.6$.

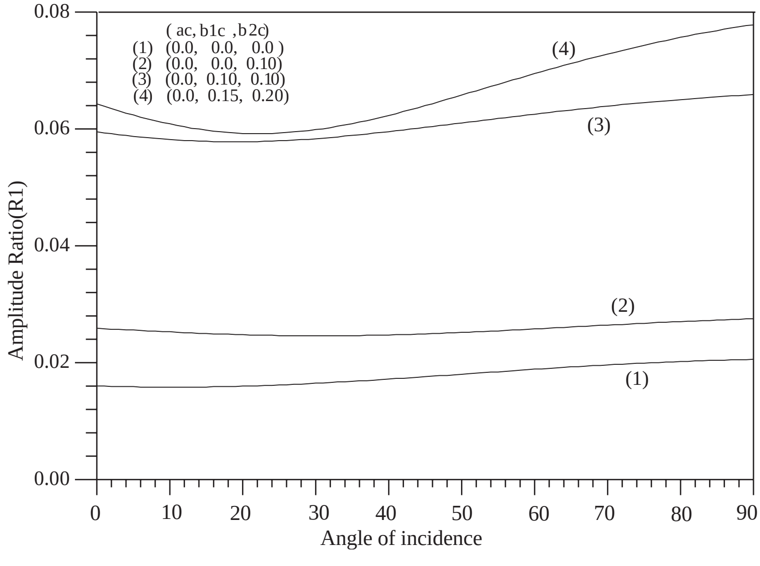

Fig. 6. Variations of R_1 with θ when $ac = 0.0, b_1c = 0.0, 0.10, 0.15$ and $b_2c = 0.0, 0.10, 0.20$.

(i) *Effect of the lateral heterogeneity*. Figs. 2–5 depict the variations of reflection and transmission coefficients for the first-order approximation of the corrugated interface with respect to the angle of incidence, when vertical heterogeneities of the media are absent and lateral heterogeneity factor vary. The effect of ac on these coefficients is clearly noticed. It is found that the values of coefficients R_1, R_1' and T_1' increases with increase of ac and θ whereas the coefficient T_1 is found to decrease with increase of angles of incidence. The magnitude of T_1' is found to be greater than those

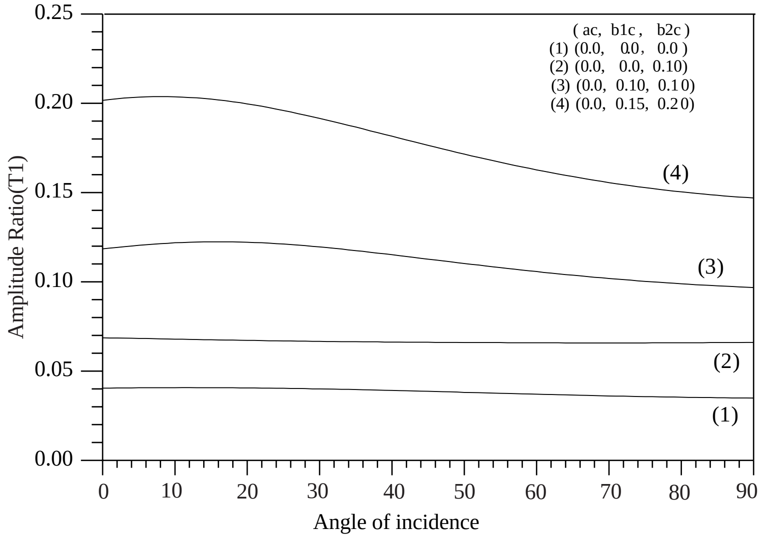

Fig. 7. Variations of T_1 with θ when $ac = 0.0$,$b_1c = 0.0$, 0.10, 0.15 and $b_2c = 0.0$, 0.10, 0.20.

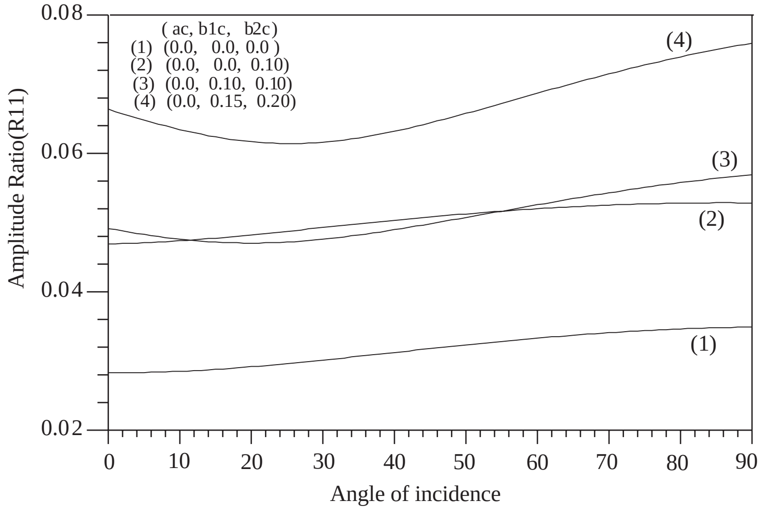

Fig. 8. Variations of R_1' with θ when $ac = 0.0$, $b_1c = 0.0$, 0.10, 0.15 and $b_2c = 0.0$, 0.10, 0.20.

of other amplitude ratios for the first order approximation of the corrugation. Also, the rate of increase or decrease of coefficients with θ is found to more significant in the absence of heterogeneities.

(ii) *Effect of the vertical heterogeneity*. Figs. 6–9 show the effect of vertical heterogeneity on R_1, T_1, R_1' and T_1' and lateral heterogeneity factor is absent in both the media. We notice from these figures that the values of these coefficients vary significantly with increase of b_1c and b_2c. The values of coefficients R_1 and T_1 increase with b_1c whereas values of coefficients R_1' and T_1' show different behaviour. The values of coefficient R_1' decreases very slowly when angle of incidence θ lies between 0° and 20° and thereafter

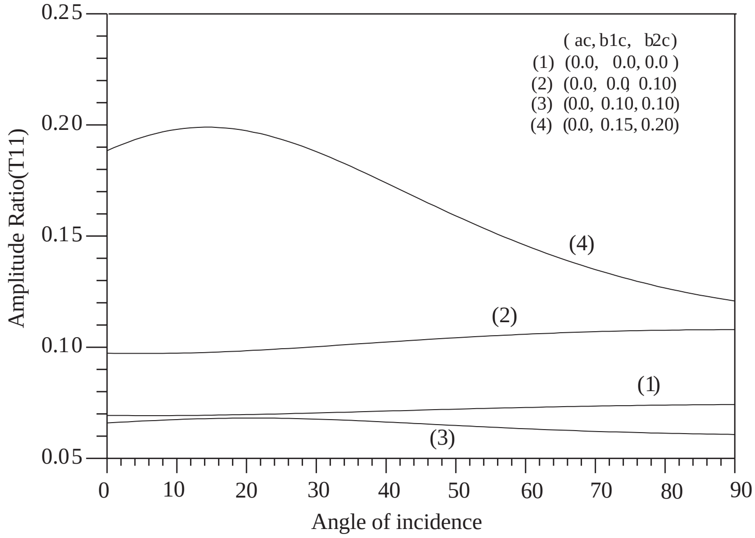

Fig. 9. Variations of T_1' with θ when $ac = 0.0$, $b_1c = 0.0, 0.10, 0.15$ and $b_2c = 0.0, 0.10, 0.20$.

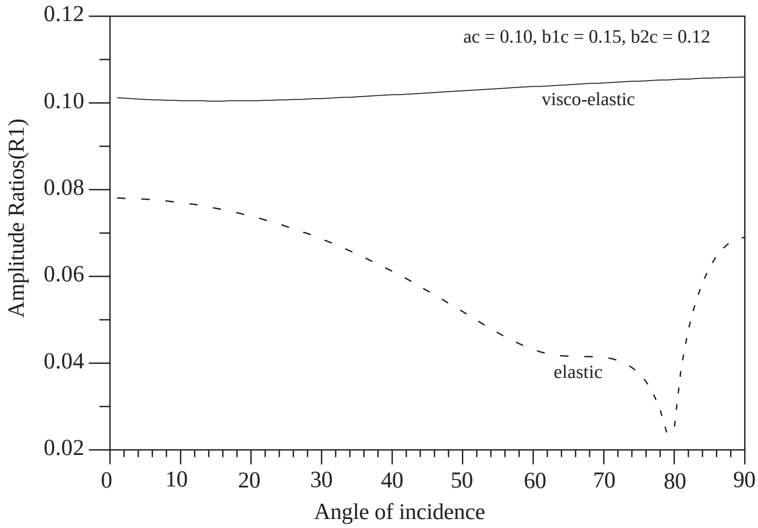

Fig. 10. Variations of R_1 with θ when $ac = 0.0$, $b_1c = 0.15$ and $b_2c = 0.12$.

increases. From Fig. 9, it is found that the values of coefficient T_1' decreases with b_1c at all angles of incidence. When only vertical heterogeneity in the medium H_2 is increased, the values of coefficients R_1, T_1, R_1' and T_1' increase at all angles of incidence. When vertical heterogeneity in both the media increases, the values of all the amplitude ratios increase at all angles of incidence. Also, in general the reflection coefficients increase with increase of θ, while transmission coefficients decrease with increase of θ in this case.

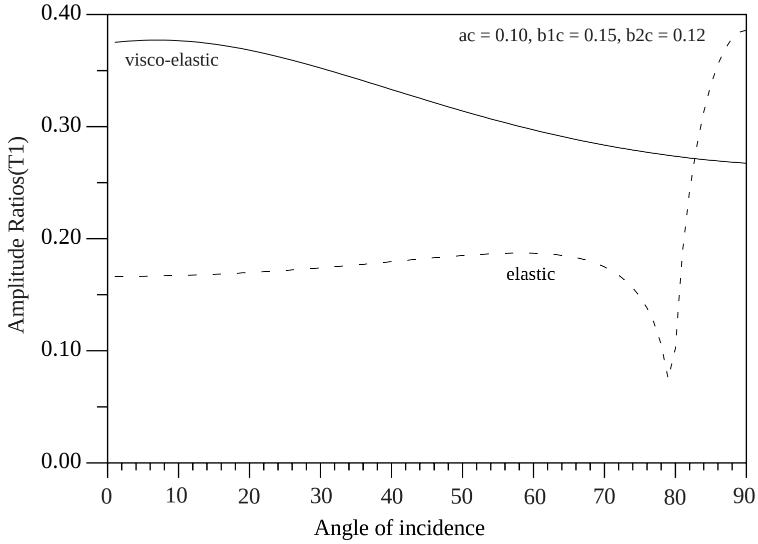

Fig. 11. Variations of T_1 with θ when $ac = 0.0$, $b_1c = 0.15$ and $b_2c = 0.12$.

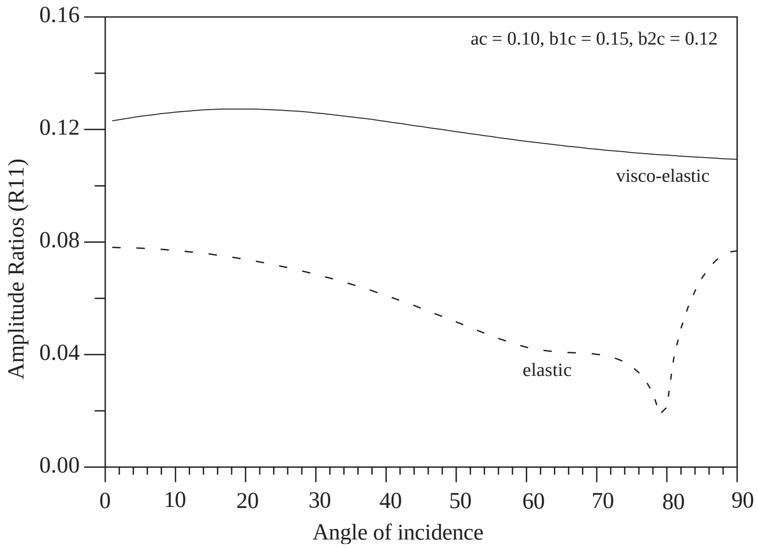

Fig. 12. Variations of R_1' with θ when $ac = 0.0$, $b_1c = 0.15$ and $b_2c = 0.12$.

(iii) *Effect of visco-elasticity*. Figs. 10–13 show the effect of visco-elasticity on various amplitude ratios when both lateral and vertical heterogeneities are present in both the media with the values $ac = 0.10$, $b_1c = 0.15$ and $b_2c = 0.12$. We notice from these figures that the values of coefficients R_1 and R_1' in viscoelastic media are larger than those in case of elastic media. The values of amplitude ratio T_1 for elastic case are less than that of viscoelastic case in the range $0° < \theta < 82°$, while in the rest of the range it is reverse. The values of T_1' for elastic case is found to be greater than that of viscoelastic case for all angles of incidence except in the neighborhood of angle $\theta = 80°$. Again, in viscoelastic media the

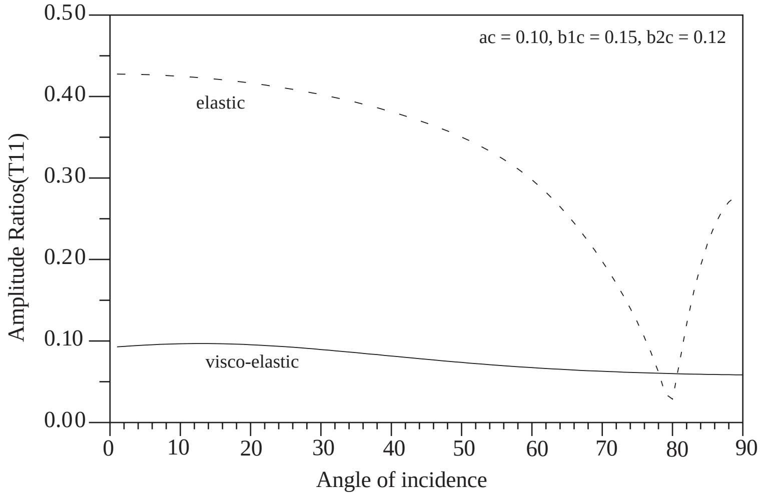

Fig. 13. Variations of T_1' with θ when $ac = 0.0$, $b_1c = 0.15$ and $b_2c = 0.12$.

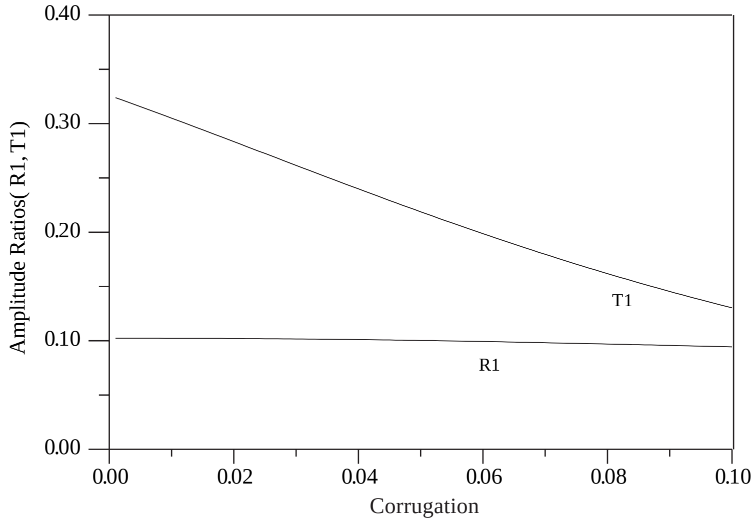

Fig. 14. Variations of R_1 and T_1 with k^*c when $ac = 0.10$, $b_1c = 0.15$, $b_2c = 0.12$.

amplitude ratio R_1 increases slowly, while all other amplitude ratios decrease with angle of incidence. On the other hand, in case of elastic media, the behaviour of coefficients R_1, R_1' and T_1' is similar to each other, while the behaviour of coefficient T_1 is that it first increases very slowly and decreases fast in the neighborhood of $\theta = 80°$ and thereafter it increases to the maximum value at $\theta = 90°$.

(iv) *Effect of corrugation*. To study the effect of corrugation parameter k^*c on the coefficients R_1, T_1, R_1' and T_1', we have computed them for very small values of k^*c with the values of parameters $ac = 0.10$, $b1c = 0.15$, $b_2c = 0.12$ and $\theta = 45°$. Figs. 14,15 show the variations of the coefficients R_1, T_1, R_1', T_1' versus k^*c. Here we notice that the coefficients T_1, T_1' and R_1' are strongly affected by the corrugation parameter k^*c, while the values of R_1 are least affected with increase of k^*c. Also, the values of T_1

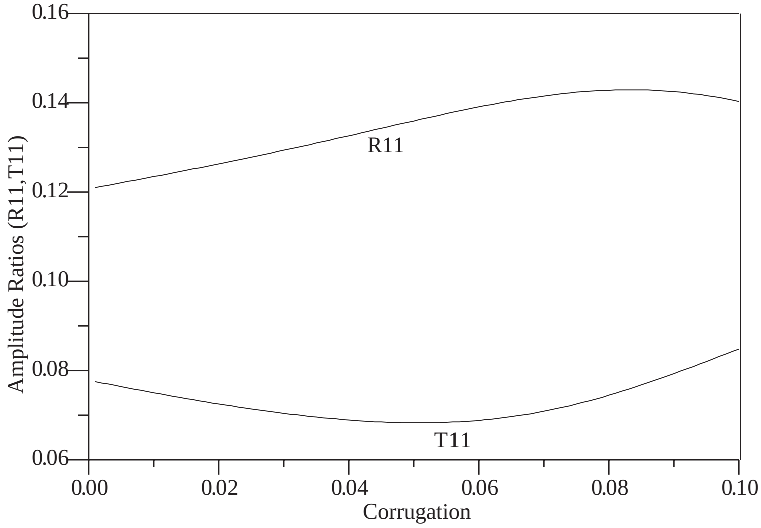

Fig. 15. Variations of R_1' and T_1' with k^*c when $ac = 0.10$, $b_1c = 0.15$, $b_2c = 0.12$.

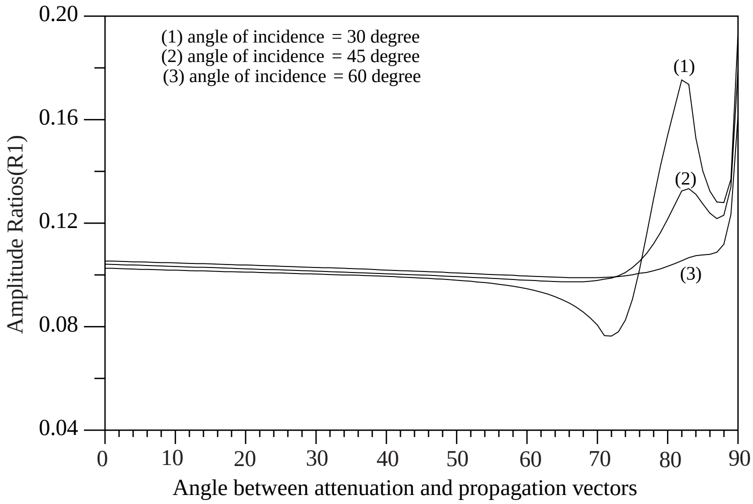

Fig. 16. Variations of R_1 with γ, when $\theta = 30°, 45°, 60°$.

decrease linearly with k^*c. We observe from these figures that the values of R_1' increase when $0 < k^*c < 0.06$ and then decrease, while the values of T_1' decrease when $0 < k^*c < 0.06$ and thereafter increases.

(v) *Effect of angle between propagation and attenuation vectors*. To study the effect of angle between propagation and attenuation vectors γ on reflection and transmission coefficients for both plane and corrugated interfaces, we fix the values of $ac = 0.10$, $b_1c = 0.15$, $b_2c = 0.12$. We notice from Figs. 16–19 that the pattern of variations of coefficients R_1, T_1, R_1', T_1' versus γ is similar. However, all coefficients have their maximum values near $\gamma = 72°$. We also noticed from these figures that with the increase of angle of incidence, each amplitude ratio decreases with the increase of γ.

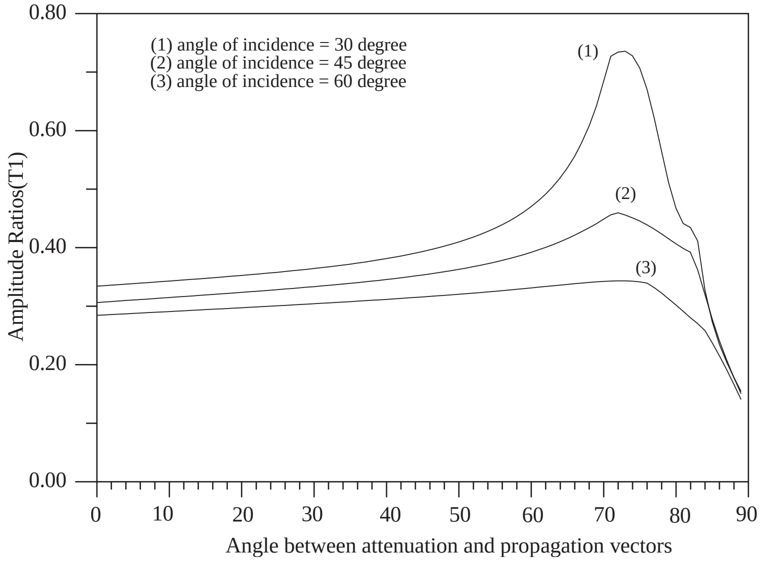

Fig. 17. Variations of T_1 with γ, when $\theta = 30°, 45°, 60°$.

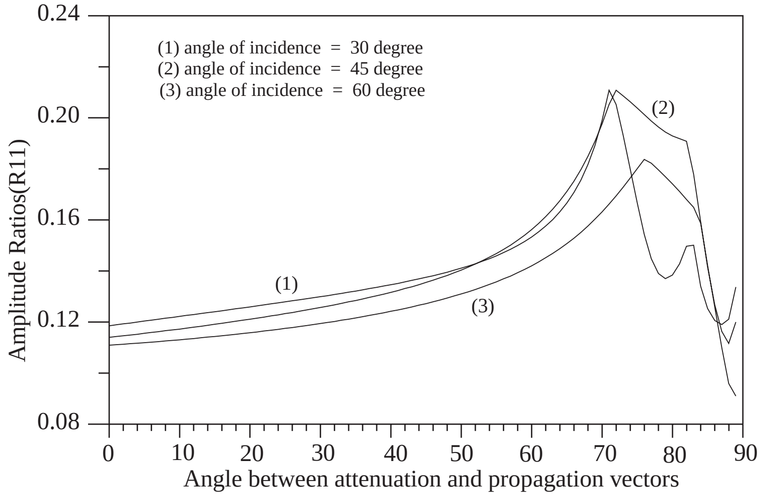

Fig. 18. Variations of R'_1 with γ, when $\theta = 30°, 45°, 60°$.

7. Conclusions and remarks

Mathematical analysis is made using Rayleigh's method of approximation to the problem of reflection and refraction coefficients of shear waves incident obliquely at a corrugated interface between two laterally and vertically heterogeneous viscoelastic solid half spaces. Formulae of the reflection and

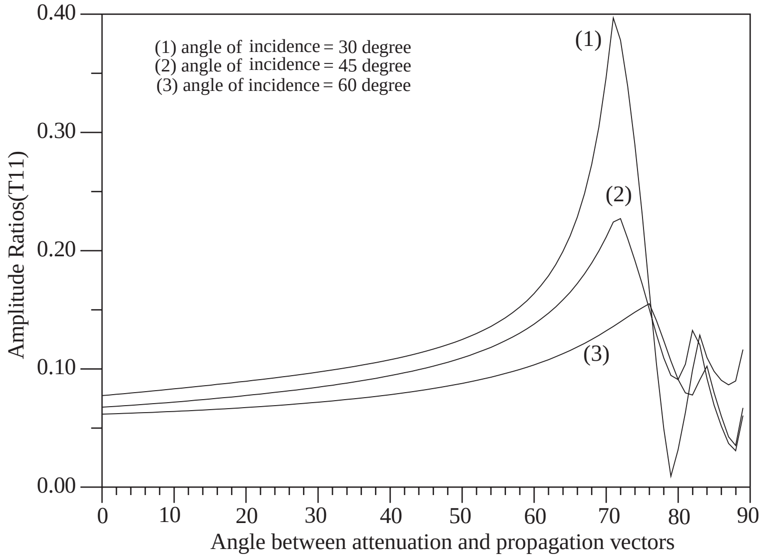

Fig. 19. Variations of T_1' with γ, when $\theta = 30°, 45°, 60°$.

transmission coefficients for first approximation of the corrugation are presented in closed form. We conclude that

1. The analytical expressions of reflection and transmission coefficients for the first order approximation of the corrugation are proportional to the amplitude of the corrugated interface. These coefficients are also the functions of angle of incidence, heterogeneity parameters and the angle between propagation and attenuation vectors of the incident wave.
2. The variations of different coefficients are found to be different with increasing values of corrugation parameter. If we remove the corrugation of the interface, then it is easy to see that all the coefficients corresponding to the first order approximation of the corrugation vanish and we are left with only reflection and refraction coefficients at the plane interface as was expected beforehand.
3. The lateral and vertical heterogeneities play an important role in changing the behaviour of reflection and transmission coefficients for the first order approximation of the corrugation. These coefficients increase with increase of lateral heterogeneity. The effect of lateral heterogeneity is found to be more dominant near normal incidence and is less dominant near the grazing incidence. In the case, when both the media are free from lateral heterogeneity and vertical heterogeneity increases, the values of reflection and transmission coefficients also found to increase.
4. A remarkable effect of lateral and vertical heterogeneity on these coefficients at a corrugated interface is found in case of viscoelastic media and that of elastic media. Reflection and transmission coefficients for the first order approximation of the corrugation are found to be significantly effected by the angle between propagation and attenuation vectors γ. Each coefficient decreases with increase of angle of incidence.
5. The results of the problems earlier discussed by Asano (1960), Singh et al. (1978), Kaushik and Chopra (1980, 1984) and Gupta (1987) have been obtained as particular cases of the present problem. Some new results have also been presented.

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